PROJECTS TO AVOID CATASTROPHES

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- Value it as such.
- I use a simple two-period example.
- "Pay" the government a fraction θ of consumption. In return, get guarantee that catastrophic drop in GDP won't happen.

Simple Two-Period Example

- Now ($t = 0$) and future ($t = T$). Under BAU, probability Λ of catastrophe on or before T. If it occurs, C_T falls by fraction ϕ:

$$C_0 \begin{array}{lll} \nearrow & (1+G_T)C_0 & \text{probability } 1-\Lambda \\ \searrow & (1-\phi)(1+G_T)C_0 & \text{probability } \Lambda \end{array}$$

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- *How* guarantor eliminates risk doesn't matter. Just want value of guarantee. Guarantor replaces lost consumption ($\phi(1+G_T)(1-\theta)C_0$) under bad outcome. Guarantor's payoffs:

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- Expected loss to guarantor is $\phi\Lambda(1+G_T)(1-\theta)C_0$.

Two-Period Example (Con't)

- Risk-free T-year bond. Risk-free rate over T years is $(1+r_f)^T - 1 \equiv R_f$. Current and future values of bond:

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- Portfolio long n units of consumption, short m units of risk-free bond. So its current value is:

$$\Phi_0 = nC_0 - mB_0 = nC_0 - \frac{100m}{1+R_f} \tag{1}$$

The value of the portfolio at time T is:

$$\Phi_T = \begin{cases} (1+G_T)nC_0 - 100m & \text{if } C_T = (1+G_T)C_0 \\ (1-\phi)(1+G_T)nC_0 - 100m & \text{if } C_T = (1-\phi)(1+G_T)C_0 \end{cases}$$

Two-Period Example (Con't)

- Choose n and m so Φ_T replicates guarantee:

$$(1 + G_T)nC_0 - 100m = 0 \tag{2}$$

$$(1 + G_T)(1 - \phi)nC_0 - 100m = -(1 - \theta)(1 + G_T)\phi C_0 \tag{3}$$

- Solve for n and m: $n = 1 - \theta$ and $m = (1 + G_T)(1 - \theta)C_0/100$
- Plug these n and m into eqn. (1):

$$\Phi_0 = (1 - \theta)C_0 \left[1 - \frac{1 + G_T}{1 + R_f}\right] . \tag{4}$$

- Assume $R_f < G_T$, so $\Phi_0 < 0$. Thus guarantor must be paid:

$$-\Phi_0 = \left[\frac{G_T - R_f}{1 + R_f}\right](1 - \theta)C_0 . \tag{5}$$

To find θ, set this payment equal to θC_0:

$$\theta = (G_T - R_f)/(1 + G_T) . \tag{6}$$

So value of guarantee is:

$$-\Phi_0 = (G_T - R_f)C_0/(1 + G_T) = \theta C_0 . \tag{7}$$

Two-Period Example (Con't)

- Effective discount rate: Equate expected PV of guarantor's payout to what he/she gets:

$$\frac{\phi\Lambda(1+R_f)C_0}{1+R_T} = -\Phi_0 = \left[\frac{G_T - R_f}{1+G_T}\right] C_0 \qquad (8)$$

Thus T-year discount rate is:

$$R_T = \frac{\phi\Lambda(1+R_f)(1+G_T)}{G_T - R_f} - 1\ . \qquad (9)$$

- Some numbers:
  - $T = 50$, $r_f = .01$, so $R_f = 1.01^{50} - 1 = 0.64$.
  - $g = .02$, so $G_T = 1.02^{50} - 1 = 1.69$.
  - Set $\Lambda = \phi = 0.3$.
  - Then $\theta = 0.39$ and value of guarantee is $0.39C_0$.
  - Also, $R_T = -0.622$. Implies annual discount rate of -0.0193.
  - If $g = .015$ so $G_T = 1.015^{50} - 1 = 1.105$, $\theta = 0.22$.

Willingness to Pay

- Would society pay so much? Need social utility function:

$$u(C) = C^{1-\eta}/(1-\eta) \ ,$$

  with $\eta > 1$. Let δ = rate of time preference. λ = annual mean arrival rate for event, so $\Lambda_T = 1 - e^{-\lambda T}$. Also, θ depends on T: $\theta_T = 1 - \left(\frac{1+r_f}{1+g}\right)^T$.

- With no guarantee, $U = u(C_0) + \mathcal{E}u(C_T)(1+\delta)^{-T}$ is

$$U_1 = \frac{C_0^{1-\eta}}{1-\eta}\left\{1 + \frac{(1+g)^{(1-\eta)T}}{(1+\delta)^T}[1 - e^{-\lambda T} + (1 - e^{-\lambda T})(1-\phi)^{1-\eta}]\right\} \tag{10}$$

  With guarantee, total utility is

$$U_2 = \left(\frac{1+r_f}{1+g}\right)^{(1-\eta)T}\frac{C_0^{1-\eta}}{1-\eta}\left\{1 + \frac{(1+g)^{(1-\eta)T}}{(1+\delta)^T}\right\} \tag{11}$$

  Should society give up $\theta_T C_0$? Only if $U_2 > U_1$. U_1 and U_2 are both negative so $U_2 > U_1$ implies $U_1/U_2 > 1$.

Ratio of Value Functions

η	T	g	λ	ϕ	θ	r_T	U_1 / U_2
2	20	0.02	0.01	0.3	.18	-.048	0.847
2	50	0.02	0.01	0.3	.39	-.014	0.634
1.5	50	0.02	0.01	0.3	.39	-.014	0.804
3	50	0.02	0.01	0.3	.39	-.014	0.392
2	20	0.015	0.01	0.3	.09	-.017	0.936
2	50	0.015	0.01	0.3	.22	-.002	0.824
2	20	0.015	0.02	0.3	.09	.013	0.960
2	50	0.015	0.02	0.3	.22	.007	0.849
2	20	0.015	0.02	0.6	.09	.048	1.097
2	50	0.015	0.02	0.6	.22	.021	1.020

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- If project were offered by a private firm, might be provided at a cost equal to WTP, so it still has a positive NPV.
- If government and society are the same, would presumably be provided at its actual cost kC_0, so its NPV is zero.

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  1. Its value as a contingent claim, θC_0
  2. Its actual cost, kC_0
  3. Its value to society, as measured by society's WTP
- Evaluating the project involves a comparison of all three of these numbers.